COMPUTING THE 2D TEMPERATURE DISTRIBUTION AND ICING DEGREE IN LOGS AT CHANGING ATMOSPHERIC TEMPERATURE IN WINTER

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Abstract:
This article presents an approach for computing the 2D temperature field, average mass temperature, and relative icing degree of logs subjected to many days and nights of continuous alternating freezing and thawing at periodically changing air temperature near them in winter. Mathematical descriptions of the periodically changing atmospheric temperature in winter and of the icing degree of the logs under the influence of that temperature have been carried out and entered in own mutually connected 2D models of logs’ freezing and thawing processes. Results from simulations with the models of the change in the 2D temperature field, average mass temperature, and relative icing degree of beech logs with a diameter of 240mm, length of 480mm, moisture content of 0.6kg·kg⁻¹, and initial temperature of 0°C during their 5 days continuous alternating freezing and thawing at changing atmospheric temperature with different initial values below −5°C and different amplitudes are shown and analyzed.

Key words: 2D models; beech logs; freezing; thawing; icing degree; atmospheric temperature.

INTRODUCTION
It is known that the duration and the energy consumption of the regimes for thermal treatment of frozen logs in the winter, which ensure their desired plasticity for the production of veneer, depend on many factors. The most influential of these factors are the initial average mass temperature of the logs and the connected with it logs’ relative icing degree (Vorreiter 1958, Kollmann and Coté 1984, Shubin 1990, Trebula and Klement 2002, Videlov 2003, Deliiski 2004, 2013b, Pervan 2009).

Results from investigations of the temperature change in subjected to thawing frozen logs only at conductive boundary conditions have been reported (Chudinov 1966, 1968, Steinhagen 1986, 1991, Khattabi and Steinhagen 1992, 1993, Deliiski 2004, 2009, 2011, Deliiski et al. 2015, Hadjiski and Deliiski 2016). Calculations of the thawing process of logs by heating them in agitated water or steam have been carried out in these publications taking into account that according to Chudinov (1966) and Kübler et al. (1973) the ice formed from the free water in the wood melts between –2°C and –1°C and the frozen bound water melts at temperatures lower than –2°C.

Results for the freezing and thawing of logs at air medium (i.e. at convective boundary conditions) are given only in (Deliiski and Tumbarkova 2016, 2019, Tumbarkova 2019). The wide experimental study of the freezing and thawing processes of cylindrical materials from diverse wood species and with various moisture contents, which has been carried out in Deliiski and Tumbarkova (2016), has shown that the freezing of the free water in the wood occurs in the range between 273.15K and 272.15K (i.e. between 0°C and –1°C) and after that from T = 272.15K (i.e. from –1°C) the freezing of the bound water in the wood begins. The experiments of these authors have also shown that the frozen free water in the wood melts between 272.15K and 273.15K (i.e. between –1°C and 0°C). Before that the gradual melting of the ice formed from the bound water in the wood ends at T = 272.15K (i.e. at –1°C). All experiments have been carried out at a curvilinear change in the temperature of the surrounding air, as follows: in a closed freezer until reaching of a temperature of about –30°C by the logs and after that in an open freezer until the logs reach room temperature.

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Coupled 2D models of the freezing and thawing processes of the logs in an air environment have been created and solved in (Deliiski and Tumbarkova 2019, Tumbarkova 2019). Using the experimentally obtained results mentioned above, the models have been verified at the same curvilinear convective boundary conditions as during the experiments. The verifying of the models at a curvilinear change of the air processing medium allows us to conduct simulative investigations by the models of the temperature and icing degree evolution in the logs subjected to the influence of periodic alternating change of the atmospheric temperature in the winter.

As a result of such a study it is possible to calculate the real values of the average mass temperature, $t_{av}$, and of the icing degree of logs, $\psi_{ice}$, depending on their dimensions, wood species, moisture content, and on the change in the air temperature near the logs during their many days staying in an open warehouse before the thermal treatment in the production of veneer. The accurate information about these parameters that cannot be measured could be used for scientifically based computing and automatic realization of the optimal, energy saving regimes for thermal treatment of each specific batch of logs.

Investigations in (Deliiski and Dzurenda 2010, Deliiski 2013b) have shown that each decrease of $t_{av}$ of the frozen beech logs by $10^\circ C$ in the beginning of their autoclave steaming causes an increase of the duration of the thermal treatment regime by about $9\%$ and of the energy needed for the logs’ thawing by approximately $16\%$. These numbers illustrate the importance of the problem for a maximum possible accurate determination of $t_{av}$ and $\psi_{ice}$ at any specific parameters of the ambient air temperature in the open warehouses where the logs stay for a long time before their thermal treatment.

The aim of this work is to realize solutions of two own coupled 2-dimensional models of the transient non-linear heat conduction in logs during their alternating freezing and thawing at convective boundary conditions with periodically changing atmospheric temperature in the winter and to study the change in the resulting 2D non-stationary temperature field, average mass temperature, and relative icing degree of beech logs above the hygroscopic range.

MATHEMATICAL MODELS OF THE 2D TEMPERATURE DISTRIBUTION IN LOGS DURING THEIR FREEZING AND SUBSEQUENT THAWING AT CONVECTIVE BOUNDARY CONDITIONS

In (Tumbarkova 2019; Deliiski and Tumbarkova 2019) the following models, which describe the 2D non-stationary temperature distribution in subjected to freezing and subsequent thawing horizontally situated logs have been created, solved, and verified:

a) During the logs’ freezing process:

\[
c_{we-fr} \cdot \rho_w \frac{\partial T(r, z, \tau)}{\partial \tau} = \lambda_{wr-fr} \left[ \frac{\partial^2 T(r, z, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, z, \tau)}{\partial r} \right] + \frac{\partial \lambda_{wr-fr}}{\partial T} \left[ \frac{\partial T(r, z, \tau)}{\partial r} \right]^2 + q_v
\]

with an initial condition

\[
T_w(r, 0, \tau) = T_w(0)
\]

and boundary conditions

\[
\begin{align*}
\frac{\partial T_w(r, 0, \tau)}{\partial r} &= \frac{-\alpha_{wp-fr}(r, 0, \tau)}{\lambda_{wp}(r, 0, \tau)} \left[ T_w(r, 0, \tau) - T_{m-fr}(\tau) \right], \\
\frac{\partial T_w(0, z, \tau)}{\partial z} &= \frac{-\alpha_{wr}(0, z, \tau)}{\lambda_{wr}(0, z, \tau)} \left[ T_w(0, z, \tau) - T_{m-fr}(\tau) \right].
\end{align*}
\]

b) During the logs’ thawing process:

\[
c_{we-thaw} \cdot \rho_w \frac{\partial T(r, z, \tau)}{\partial \tau} = \lambda_{wr-nfr} \left[ \frac{\partial^2 T(r, z, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, z, \tau)}{\partial r} \right] + \frac{\partial \lambda_{wr-nfr}}{\partial T} \left[ \frac{\partial T(r, z, \tau)}{\partial r} \right]^2 + q_v
\]

with an initial condition

\[
T_w(r, z, 0) = T_w(r, z, \tau_{fre})
\]

and boundary conditions
\[
\frac{\partial T_w(r,0,\tau)}{\partial r} = -\frac{c_{wp-thaw}(r,0,\tau)}{\lambda_{wp}(r,0,\tau)} \left[ T_w(r,0,\tau) - T_{m-thaw}(\tau) \right],
\]
(7)

\[
\frac{\partial T_w(0,z,\tau)}{\partial z} = -\frac{c_{wr-thaw}(0,z,\tau)}{\lambda_{wr}(0,z,\tau)} \left[ T_w(0,z,\tau) - T_{m-thaw}(\tau) \right],
\]
(8)

where: \(c_{wp-thaw}\) and \(c_{wp-thaw}\) are the effective specific heat capacities of the wood during the freezing and thawing of logs respectively in temperature ranges, in which the free and the bound water crystalize or melt (Chudinov 1968, Deliiski 2009, 2011, 2013b, Deliiski and Tumbarkova 2019), in J·kg\(^{-1}\)·K\(^{-1}\);

\(\lambda_{wp-thaw}\) and \(\lambda_{wp-thaw}\) – thermal conductivities of the frozen wood in radial and longitudinal direction respectively, in W·m\(^{-1}\)·K\(^{-1}\);

\(\lambda_{wr-thaw}\) and \(\lambda_{wr-thaw}\) – thermal conductivities of the non-frozen wood in radial and longitudinal direction respectively, in W·m\(^{-1}\)·K\(^{-1}\);

\(\rho_w\) – wood density, in kg·m\(^{-3}\);

\(r\) – coordinate of the separate points along the log’s radius, in m;

\(z\) – coordinate of the separate points along the log’s length, in m;

\(\tau\) – time, in s;

\(\tau_{fre}\) – terminal time of the freezing process, in s;

\(T_w\) – temperature of the wood, in K;

\(T_{w0}\) – initial average mass temperature of the subjected to freezing log, in K;

\(T_w(r,z,0)\) – temperature of all points in the log’s volume at the beginning of the freezing or thawing process, in K;

\(T_w(r,0,\tau)\) – temperature of all points on the log’s frontal surface during the freezing or thawing process, in K;

\(T_w(0,z,\tau)\) – temperature of all points on log’s cylindrical surface during the freezing or thawing process, in K;

\(T_{m-fr}\) and \(T_{m-thaw}\) – temperature of the surrounding air environment during the log’s freezing and thawing respectively, in K;

\(q_v\) – internal heat source in the log’s volume caused by the release of the latent heat of both the free and bound water during their crystallization (Efimov 1985, Pahi 2010, https://www.physics.info/heat-latent/), in W·m\(^{-3}\);

\(\alpha_{wr-fr}\) and \(\alpha_{wr-thaw}\) – convective heat transfer coefficients between the log’s surfaces and ambient air environment in radial and longitudinal direction respectively during the freezing, in W·m\(^{-2}\)·K\(^{-1}\);

\(\alpha_{wr-thaw}\) and \(\alpha_{wp-thaw}\) – convective heat transfer coefficients between the log’s surfaces and ambient air in radial and longitudinal direction respectively during the thawing process, in W·m\(^{-2}\)·K\(^{-1}\).

On Fig. 1 the positioning of the coordinate axis \(r\) and \(z\), and also of 4 representative knots of the calculation mesh is shown, which was used below for the numerical solving of the models (1) to (8).
For the computing of the current amount of non-frozen water in the logs and of their icing degree, synchronously with the solving of the models, the average mass temperature of the logs, $T_{avg}$, for each moment of their alternating freezing and thawing is calculated according to the equation

$$T_{avg} = \frac{1}{S} \int_{S} T(r, z, \tau) dS,$$

where: $S$ is the area of $\frac{1}{4}$ of the logs’ longitudinal section, on which the calculation mesh for the numerical solving of the models is built, in m$^2$.

Mathematical descriptions of all variables, which participate in eqs. (1) to (8), depending on the specific influencing factors have been given in Deliiski (2004, 2009, 2011, 2013a, 2013b) and in (Deliiski and Tumbarkova 2019, Tumbarkova 2019).

For the calculation of the heat transfer coefficients of the beech logs considered below the following experimentally verified equations were used (Telegin et al. 2002, Tumbarkova 2019):

- in the radial direction on the cylindrical surface of the logs:

$$\alpha_{wr-fr} = 1.123 (T(0, z, \tau) - T_m(\tau))^{0.46}, \quad [W\cdot m^{-2}\cdot K^{-1}]$$

$$\alpha_{wr-thaw} = 1.123 (T(0, z, \tau) - T_m(\tau))^{0.26}, \quad [W\cdot m^{-2}\cdot K^{-1}]$$

- in the longitudinal direction on the frontal surface of the logs:

$$\alpha_{wp-fr} = 2.56 (T(r, 0, \tau) - T_m(\tau))^{0.46}, \quad [W\cdot m^{-2}\cdot K^{-1}]$$

$$\alpha_{wp-thaw} = 2.56 (T(r, 0, \tau) - T_m(\tau))^{0.26}, \quad [W\cdot m^{-2}\cdot K^{-1}]$$

**MATHEMATICAL DESCRIPTION OF THE PERIODICALLY CHANGING ATMOSPHERIC TEMPERATURE**

For numerical solving of the models (1) ÷ (4) and (5) ÷ (8) for the cases of the considered in this work alternating logs’ freezing and thawing processes it is needed to have a mathematical description of the temperature of the air medium in winter near the logs, $T_{m-fr}$ and $T_{m-thaw}$.

The periodic change of the ambient air temperature $T_m$ at a constant value of its amplitude $T_{ma}$ can be described by the equation (Guzenda and Ganowicz 1986, Deliiski 1988, Olek and Guzenda 1995):

$$T_m = T_{m0} + (T_{ma} - T_{m0}) \cdot \sin(\omega \cdot \tau), \quad [K]$$

where:

- $T_{m0}$ is the initial value of the periodically changing atmospheric temperature, in K;
- $T_{ma}$ – amplitude’s value of $T_m$, in K;
- $\omega$ – angular frequency of the change in $T_m$, which is equal to

$$\omega = \frac{2\pi}{\tau_{per}}, \quad [s^{-1}]$$

where: $\tau_{per}$ is the period of change in $T_m$, in s. For the precise solving of tasks with the participation of $\omega$ it is needed to use $\pi = 3.14159$.

For a periodic change of the air temperature during one day and night, i.e. at $\tau_{per} = 1 \text{d} = 24 \text{h} = 86400$ s, according to eq. (15) it is obtained that

$$\omega = \frac{2\pi}{\tau_{per}} = \frac{2 \cdot 3.14159}{86400} = 7.2722 \cdot 10^{-3} \text{ s}^{-1}.$$

The change of the values of $T_{m0}$ and $T_{ma}$ in the beginning of each subsequent period compared to their initial values, $T_{m0-in}$ and $T_{ma-in}$, respectively, can be described by the equation

$$T_m = T_{m0-in} \cdot (1 \pm K_{m0} \cdot \tau) + (T_{ma-in} - T_{m0-in}) \cdot (1 \pm K_{ma} \cdot \tau) \cdot \sin(\omega \cdot \tau), \quad [K]$$

where: $T_{m0-in}$ is the initial value of $T_m$, i.e. the value of $T_m$ in the beginning of its 1st period of change, in K;

$T_{ma-in}$ – initial amplitude’s value of $T_m$, i.e. the value of $T_{ma}$ in the first half of the 1st period of $T_m$ change, in K;
$K_{m0}$ and $K_{ma}$ – coefficients of change in $T_{m0-in}$ and $T_{ma-in}$ during the periodically change of $T_m$. These coefficients can be calculated according to the following equations:

$$K_{m0} = \frac{\Delta T_{m0-per}}{\tau_{per}}, \quad [s^{-1}] \quad (17)$$

$$K_{ma} = \frac{\Delta T_{ma-per}}{\tau_{per}}, \quad [s^{-1}] \quad (18)$$

where: $\Delta T_{m0-per}$ is the difference between the initial values of $T_m$ in its two adjacent periods, in K;

$\Delta T_{ma-per}$ – difference between the amplitude values of $T_m$ in its two adjacent periods, in K.

The signs “+” and “−” in the right-hand side of eq. (16) are used when the values of $T_{m0-in}$ and $T_{ma-in}$ increase or decrease respectively during the periodic change in $T_m$.

MATHEMATICAL DESCRIPTION OF THE CHANGE IN THE RELATIVE ICING DEGREE OF LOGS

For the solving of the model (1) ÷ (4) and for our investigations below it is needed to have a mathematical description of the change in the relative icing degree of logs, $\psi_{ice}$.

As it was mentioned above, the results in (Deliiski and Tumbarkova 2016) from experimental research of the freezing process of logs from some wood species at different moisture contents show that the free water in the wood crystallizes in the temperature range between 273.15K and 272.15K (i.e. between 0°C and –1°C). After the whole amount of the free water in the wood freezes, a freezing of the bound water in the wood starts. The quantity of frozen bound water increases with the decrease of the temperature below –1°C, but even during extremely small climatic temperatures on earth, a definite part of it remains in a non-frozen state (Chudinov 1966, 1968, Topgaard and Söderman 2002).

The relative icing degree of the wood, $\psi_{ice}$, can be expressed as a relationship between the weight of the ice in 1kg wood and the total weight of the ice and the non-frozen water in 1kg wood (Chudinov 1968), i.e.

$$\psi_{ice} = \frac{m_{ice}}{m_{ice} + m_{nfw}} = \frac{u - u_{nfw}}{u - u_{nfw} + u_{nfw}} = 1 - \frac{u_{nfw}}{u}, \quad [-] \quad (19)$$

where: $m_{ice}$ is the weight of the ice in 1 kg wood, in kg;

$m_{nfw}$ – weight of the non-frozen water in 1 kg wood, in kg;

$u$ – moisture content of the wood, in kg.kg$^{-1}$;

$u_{nfw}$ – amount of the bound water in the wood in non-frozen state at a given temperature below 272.15 K, which in our case is equal to (Deliiski 2011, 2013a, 2013b):

$$u_{nfw} = 0.12 + \left(\frac{272.15}{u_{fp}} - 0.12\right) \exp\left[0.0567(T_{avg} - 272.15)\right] \text{ at } T_{avg} \leq 272.15 \text{ K}, \quad [\text{kg} \cdot \text{kg}^{-1}] \quad (20)$$

where: $u_{fp}^{272.15}$ is the fiber saturation point of the wood species at $T = 272.15$ K, i.e at $t = -1$ °C. As it was pointed above, at this temperature the freezing of the whole amount of the free water has been completed and the freezing of the bound water in the wood starts (Deliiski and Tumbarkova 2016).

Using experimental data of Stamm (1964), Deliiski (2013b) the following equation for calculation of the fiber saturation point of the wood species has been suggested, depending on $T$:

$$u_{fp} = u_{fp}^{293.15} - 0.001(T - 293.15), \quad [\text{kg} \cdot \text{kg}^{-1}] \quad (21)$$

where: $u_{fp}^{293.15}$ is the standardized fiber saturation point at $T = 293.15$ K, i.e. at $t = 20$°C.

According to eq. (17), the fiber saturation point of the wood species at $T = 272.15$ K is equal to
\[ u_{\text{fp}}^{272.15} = u_{\text{fp}}^{293.15} - 0.001(272.15 - 293.15) = u_{\text{fp}}^{293.15} + 0.021. \]  

(22)

RESULTS AND DISCUSSION

Computing the 2D temperature change in logs during their periodic freezing and thawing

The mathematical descriptions of the thermo-physical properties and icing degree of logs considered above, and also of the periodically changing atmospheric temperature in winter were entered in the coupled mathematical models (1) + (4) and (5) + (8). For the numerical solving of the models, a software program was prepared in the calculation environment of Visual FORTRAN developed by Microsoft. Using the program computations were carried out for the determination of the 2D change of \( t \) in three beech logs named below as Log 1, Log 2, and Log 3. The logs were with a diameter \( D = 240 \text{mm} \), length \( L = 480 \text{mm} \), basic density \( \rho_b = 560 \text{kg} \cdot \text{m}^{-3} \), initial temperature \( t_{m0} = 0^\circ \text{C} \), moisture content above the hygroscopic range \( u = 0.6 \text{kg} \cdot \text{kg}^{-1} \), and standardized fiber saturation point \( u_{\text{fp}}^{293.15} = 0.31 \text{kg} \cdot \text{kg}^{-1} \) (Videlov 2003).

Three options of 120h continuous alternating freezing and thawing of the logs have been simulated with the models as follows:

• for Log 1: at constant values of the initial air temperature \( T_{m0} = 268.15 \text{ K} \) (i.e. \( t_{m0} = -5 \text{ }^\circ \text{C} \)) and of the air temperature \( T_{ma} = 288.15 \text{ K} \) (i.e. \( t_{ma} = T_{ma} - T_{m0} = 20 \text{ }^\circ \text{C} \));
• for Log 2: at gradual increasing of the value of \( t_{m0-in} = -5 \text{ }^\circ \text{C} \) by 1 \text{ }^\circ \text{C}/\text{d} \) and gradual decreasing of the value of \( t_{ma-in} = 20 \text{ }^\circ \text{C} \) by 2 \text{ }^\circ \text{C}/\text{d} \);
• for Log 3: at gradual decreasing of the values of \( t_{m0-in} = -5 \text{ }^\circ \text{C} \) and \( t_{ma-in} = 20 \text{ }^\circ \text{C} \) by 2 \text{ }^\circ \text{C}/\text{d}.

For the calculations of Log 3 the following values of the coefficients \( K_{m0} \) and \( K_{ma} \) in eq. (16) were used:

\[ K_{m0} = \frac{\Delta T_{m0-in}}{T_{ma-in} - T_{m0-in}} = \frac{2}{288.15 - 268.15} = \frac{268.15}{86400} = 3.157 \times 10^{-6} \text{ }^\circ \text{C}^{-1}. \]

According to eq. (22), at \( u_{\text{fp}}^{293.15} = 0.31 \text{kg} \cdot \text{kg}^{-1} \) the studied logs contain a maximum possible bound water, equal to \( u_{\text{fp}}^{272.15} = 0.331 \text{kg} \cdot \text{kg}^{-1} \). This means that at \( u = 0.6 \text{kg} \cdot \text{kg}^{-1} \) the logs contain a free water, equal to \( u - u_{\text{fp}}^{272.15} = 0.6 - 0.331 = 0.269 \text{kg} \cdot \text{kg}^{-1} \).

The models were solved without any simplification using an explicit form of the finite-difference method (Deliiski 2011, 2013b, Deliiski and Tumbarkova 2019). For this purpose the calculation mesh was built on \( 1/4 \) of the longitudinal section of the log due to the circumstance that this \( 1/4 \) is mirror symmetrical towards the remaining \( 3/4 \) of the same section (refer to Fig. 1).

Figures 2, 3, and 4 present the calculated change in the temperature of the ambient air, \( t_m \), temperature on the logs’ surface, \( t_s \), average mass temperature of the logs, \( t_{avg} \), and \( t \) of 4 representative points in Log 1, Log 2, and Log 3 respectively during their continuous 5 days and nights (i.e. 120h) alternating freezing and thawing under the described above conditions of the atmospheric temperature influence.

The coordinates of the representative points of the logs are equal to: Point 1 with temperature \( t_1: r = R/4 = 30 \text{mm} \) and \( z = L/4 = 120 \text{mm} \); Point 2 with \( t_2: r = R/2 = 60 \text{mm} \) and \( z = L/4 = 120 \text{mm} \); Point 3 with \( t_3: r = 3R/4 = 90 \text{mm} \) and \( z = 3L/8 = 180 \text{mm} \); Point 4 with \( t_4: r = R = 120 \text{mm} \) and \( z = L/2 = 240 \text{mm} \). These coordinates of the points allow for determination and analyzing of the 2D temperature distribution in logs during their periodic alternating freezing and thawing.

On Fig. 2 it can be seen that at constant values of \( t_{m0} \) and \( t_{ma} \) after 72\textsuperscript{nd} h, i.e. after the 3\textsuperscript{rd} period of \( t_m \), a periodical change in the log’s temperature with practically constant amplitudes for the separate points is coming. As far as the point is distanced from the logs’ surfaces that much smaller is the amplitude of the periodical change of the temperature in that point. The amplitudes of \( t_m, t_s \), and \( t \) in the separate representative points after the 3\textsuperscript{rd} period are equal to as follows: \( t_{ma} = 20.0 \text{ }^\circ \text{C}, t_{sa} = 9.5 \text{ }^\circ \text{C}, t_{sa} = 8.2 \text{ }^\circ \text{C}, t_{sa} = 7.2 \text{ }^\circ \text{C}, t_{sa} = 6.5 \text{ }^\circ \text{C}, t_{sa} = 6.0 \text{ }^\circ \text{C} \).

When \( t_{m0} \) increases by \( 1 \text{ }^\circ \text{C}/\text{d} \) and synchronously with this \( t_{ma} \) decreases by 2\textsuperscript{nd} C/d during the time, the amplitudes of \( t \) in the separate points decrease and during the second half of the last 5\textsuperscript{th} period of \( t_m \) they are equal to: \( t_{sa} = 6.7 \text{ }^\circ \text{C}, t_{sa} = 6.2 \text{ }^\circ \text{C}, t_{sa} = 5.8 \text{ }^\circ \text{C}, t_{sa} = 5.5 \text{ }^\circ \text{C} \) (Fig. 3).

When \( t_{ma} \) and \( t_{ma} \) decrease during the time, the amplitudes of \( t \) in the separate points also gradually decrease (Fig. 4). At \( t_{m0} = -5 \text{ }^\circ \text{C} - 2 \text{ }^\circ \text{C}/\text{d} \) and \( t_{ma} = 20 \text{ }^\circ \text{C} - 2 \text{ }^\circ \text{C}/\text{d} \), the amplitudes of \( t_m, t_s \), and \( t \) in the representative points during the second half of the last 5\textsuperscript{th} period of \( t_m \) are equal to: \( t_{sa} = 12.0 \text{ }^\circ \text{C}, t_{sa} = 7.3 \text{ }^\circ \text{C}, t_{sa} = 6.7 \text{ }^\circ \text{C}, t_{sa} = 6.2 \text{ }^\circ \text{C}, t_{sa} = 5.8 \text{ }^\circ \text{C}, t_{sa} = 5.5 \text{ }^\circ \text{C} \).

The average mass temperature of the logs, \( t_{avg} \), at 120\textsuperscript{th} h is equal to: \( -13.56 \text{ }^\circ \text{C} \) for Log 1, \( -2.75 \text{ }^\circ \text{C} \) for Log 2, and \( -19.30 \text{ }^\circ \text{C} \) for Log 3.
Computing the icing degree of logs during their periodic freezing and thawing

Synchronously with the computing of the 2D change of $t$ in the longitudinal sections of the studied logs, calculations of the change in their icing degree $\Psi_{\text{ice}}$ during the alternating freezing and thawing has been carried out (Fig. 5).

As it is seen on Fig. 5, during the first 2h of the 1st period of $t_m$ the icing degree $\Psi_{\text{ice}}$ increases from 0 to 0.07 due to the crystallizing of the free water only in some peripheral layers of the logs. From 2nd to 12th hour of the 1st period $\Psi_{\text{ice}}$ is equal to 0 because then the temperature of the entire volume of the logs is higher than 0°C and both the free and bound water in them are in a liquid state.

During the second half of the 1st period of $t_m$ the icing degree $\Psi_{\text{ice}}$ increases from 0 to 0.55 for Log 1, from 0 to 0.51 for Log 2, and from 0 to 0.56 for Log 3. These values of $\Psi_{\text{ice}}$ mean that 55% (i.e. 0.33 kg·kg$^{-1}$) from the whole amount of the moisture content of 0.6 kg·kg$^{-1}$ for Log 1, 51% (i.e. 0.306 kg·kg$^{-1}$ for Log 2, and 56% (i.e. approximately 0.34 kg·kg$^{-1}$) for Log 3 are in a frozen state.
After the 1st period of \( t_m \) the change in \( \Psi_{\text{ice}} \) is periodical and during the last 5th period it is in the range from 0.38 to 0.64 for Log 1, from 0.39 to 0.53 for Log 2, and from 0.56 to 0.68 for Log 3.

![Graph showing the change in \( t_m, t_s, t_{avg}, \) and \( t \) in 4 points of the Log 3 during its 120 h freezing and thawing.](image1)

*Fig. 4.* Change in \( t_m, t_s, t_{avg}, \) and \( t \) in 4 points of the Log 3 during its 120 h freezing and thawing.

At the end of the 5th period of \( t_m \) the icing degree \( \Psi_{\text{ice}} = 0.627 \) for Log 1, \( \Psi_{\text{ice}} = 0.482 \) for Log 2, and \( \Psi_{\text{ice}} = 0.675 \) for Log 3. These values of \( \Psi_{\text{ice}} \) mean that 62.7% (i.e. 0.376kg·kg\(^{-1}\)) for Log 1, 48.2% (i.e. 0.289kg·kg\(^{-1}\)) for Log 2, and 67.5% (i.e. 0.405kg·kg\(^{-1}\)) for Log 3 from the whole amount of the logs’ moisture content of 0.6kg·kg\(^{-1}\) are then in a frozen state.

The rest amounts of \( u \), i.e. 37.3% (i.e. 0.224kg·kg\(^{-1}\)) for Log 1, 51.8% (i.e. 0.311kg·kg\(^{-1}\)) for Log 2, and 32.5% (i.e. 0.195kg·kg\(^{-1}\)) for Log 3 are in a liquid state at the end of 120h periodic freezing and thawing of the studied beech logs.

**CONCLUSIONS**

This article presents an approach for computing the 2D temperature field, average mass temperature, and relative icing degree in logs subjected to many days and nights continuous alternating freezing and thawing at periodically changing air temperature near them in winter.
Mathematical descriptions of the periodically changing atmospheric temperature in winter and of the icing degree of the logs under influence of that temperature have been carried out and entered in coupled 2D models of logs’ freezing and thawing processes.

It has been determined that at constant values of $t_m^0$ and $t_m^a$ after the 72nd h, i.e. after the 3rd period of $t_m$, a periodical change in the log’s temperature with constant amplitudes for the separate representative points is coming. When $t_m^0$ and $t_m^a$ decrease during the time the amplitudes of the temperature in the separate points also gradually decrease. As far the given point is distanced from the logs’ surfaces that smaller the amplitude of the periodically change of the temperature in that point is. It has been calculated that at the end of the continuous 120h periodic freezing and thawing of the studied logs their average mass temperature is equal to $–13.56\degree C$ for Log 1, to $–2.75\degree C$ for Log 2, and to $–19.30\degree C$ for Log 3. It was calculated that at these values of $t_{avg}$ the wood still contains non-frozen water, whose amount is equal to $0.224\text{kg·kg}^{-1}$ for Log 1, to $0.311\text{kg·kg}^{-1}$ for Log 2, and to $0.195\text{kg·kg}^{-1}$ for Log 3.

It has been computed that after 120h of the studied alternating processes the relative icing degree of the logs reach the following values: $\Psi_{\text{ice}} = 0.627$ for Log 1, $\Psi_{\text{ice}} = 0.482$ for Log 2, and $\Psi_{\text{ice}} = 0.675$ for Log 3. This means that from the total amount of free and bound water in the logs, equal to $0.6\text{kg·kg}^{-1}$, in a frozen state are only $0.627 \times 0.6 = 0.376\text{kg·kg}^{-1}$ in Log 1, $0.482 \times 0.6 = 0.289\text{kg·kg}^{-1}$ in Log 2, and $0.675 \times 0.6 = 0.405\text{kg·kg}^{-1}$ in Log 3.

The presented approach for the computation of the 2D temperature field, average mass temperature, and icing degree of the logs at periodically changing atmospheric temperature in winter can help for accurate determination of the immeasurable by sensors initial temperature of the logs before heating them in agitated water or steam, depending on the duration of the logs’ staying in an open warehouse.

The solution of the models with the presented convective boundary conditions allows for the computing also of various energy characteristics of logs from diverse wood species with different dimensions for each moment of their alternating freezing and thawing at periodically changing air temperature with specific parameters.

The suggested approach and the results from the solutions of the models can be used for development and implementation of advanced systems for model-based control (Hadjiski et al. 2019) of the logs' thermal treatment processes.

REFERENCES


