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MECHANIC-MATHEMATICAL MODEL FOR INVESTIGATIONS OF THE FORCED SPATIAL VIBRATIONS OF WOOD SHAPER AND ITS SPINDLE, CAUSED BY UNBALANCE OF THE CUTTING TOOL

Georgi VUKOV
University of Forestry – Sofia, Faculty of Forest Industry
Blvd. Kliment Ohridski 10, 1797 Sofia, Bulgaria
Tel: 359 2 91907/285, E-mail: givukov@ltu.bg

Zhivko GOCHEV
University of Forestry – Sofia, Faculty of Forest Industry
Blvd. Kliment Ohridski 10, 1797 Sofia, Bulgaria
E-mail: zhivkog@ltu.bg

Valentin SLAVOV
University of Chemical Technology and Metallurgy – Sofia, Faculty of Metallurgy
Blvd. Kliment Ohridski 8, 1756 Sofia, Bulgaria
E-mail: valslav@abv.bg

Pavlin VICHEV
University of Forestry – Sofia, Faculty of Forest Industry
Blvd. Kliment Ohridski 10, 1797 Sofia, Bulgaria
E-mail: p_vitchev@abv.bg

Valentin ATANASOV
University of Forestry – Sofia, Faculty of Forest Industry
Blvd. Kliment Ohridski 10, 1797 Sofia, Bulgaria
E-mail: vatanasov_2000@yahoo.com

Abstract:
A mechanic - mathematical model of wood shaper and its spindle, developed by the authors, is presented in this work. The model provides the opportunity to explore the forced space vibrations of this type of machinery, caused by unbalance of the cutting tool. It takes into account the characteristics in the construction of wood shapers. In this model the wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic and damping elements with each other and with the motionless floor. The model takes into account the necessary mass, inertia, elastic and damping properties of the elements of the considered system. It includes all needed geometric parameters of this system. A necessary system of matrix differential equations is compiled and analytical solutions are presented. Numerical solutions can be obtained with their help by using the parameters of a specific machine.

Key words: *wood shapers; forced vibrations.*

INTRODUCTION

The high productivity of the operation of wood shapers, and the precision and quality of their production are the main requirements to them. The main factors determining the implementation of these requirements are the status and serviceability of the cutting tool of these machines (Filipov 1977, Grigorov 1985). Some fixed regulations to their geometry, materials for production, ways of assembly of the instruments to the shaft and etc, are required. The practice in the exploitation of wood shapers indicates that one of the common problems in their use is the presence of unbalance (disbalance) of their cutting tools. The causes for rising of the unbalance are: wrong or incorrect installation of the instrument on the shaft; uneven wear or damage of the tool; accumulation of superposition in separate parts of the instrument; occurrence of gaps and the etc. The influence of the unbalance of the cutting tool on the machine has to be studied in order to be done some research. The machine can be seen as a mechanical vibrating system with known characteristics in this research (Amirouche 2006, Angelov and Slavov 2010, Coutinho 2010).

It is known that there may be few cases of unbalance. The cutting tool is statically unbalanced if its center of gravity does not lie on the axis of rotation, but it is the main inertial axis. The static unbalance can be detected with static tests. The cutting tool is dynamically unbalanced if its center of gravity lies on the axis of rotation, but it is not the main inertial axis. The dynamic unbalance cannot be detected with static tests. The cutting tool can be statically and dynamically unbalanced at the same time, of course. This paper examines the static imbalance of the cutting tool.

The presence of unbalance of the cutting tool generates variable loads during the operation of the wood shapers. These loads are transmitted to the spindle and by its two bearing units reach to the machine's body. On the other hand, vibrations, generated by other elements of the machine, reach the spindle and the cutter back through the bearing units. It is clear that the characteristics of the bearing units (stiffness, damping properties, etc.) are important for the interaction between the spindle with the cutter and the machine's body, and consequently, for the work of the whole machine.

To sum up, it is clear that the wood shaper and its spindle are appropriate to be regarded as rigid bodies. They are connected by elastic and damping elements with each other and with the motionless floor. These elastic and damping elements are four vibration isolators between the machine and the floor, as well as both bearing units of the spindle. This approach has been used in previous works of the authors. The free space vibrations are examined (Vukov et. al. 2016) and the natural frequencies and mode shapes of the free spatial vibrations of real wood shaper are investigated numerically (Vukov et. al. 2016). The free damped spatial vibrations are discussed (Vukov et. al. 2016) and some numerical studies are conducted (Vukov et. al. 2016).

OBJECTIVE

The main objective of this study is to develop a mechanic - mathematical model of a wood shaper and its spindle, which gives the opportunity for exploration the forced space vibrations of this type of machinery, caused by unbalance of the cutting tool. The model refers to wood shapers with lower placement of the spindle. The model renders in account the construction's characteristics of this class of wood shapers. The developed model allows making numerical investigations by using parameters of real machines.

MATERIAL, METHOD, EQUIPMENT

The wood shapers with lower placement of the spindle that is commonly used in the practice of the forestry industry (Filipov 1977, Obreshkov 1996) is examined in the proposed study. Analysis of their construction shows the strong influence of the unbalance of the cutting tool on the functioning of the whole machine. Fig. 1 shows the general view, and Fig. 2 – the spindle with its bearing units and fitted cutter.

Fig. 1.
Wood shaper – general view.

Fig. 2.
Spindle with bearing units and with cutter.

The wood shaper and its spindle are regarded as rigid bodies in the following discussions. They are connected by elastic and damping elements with each other and with the motionless floor. These elastic and damping elements are four vibration isolators between the machine and the floor, as well as the two bearing units of the spindle. The static unbalance of the cutting tool, which is frequent in practice, is considered in this study. This unbalance is modeled with the introduction of a centrifugal force acting on the cutter. The value of centrifugal force is determined by the magnitude of the unbalanced mass, the eccentricity (the distance from the axis of rotation to the center of gravity of the tool) and the square of the angular velocity.

A mechanic - mathematical model of wood shapers with lower spindle is built for studying its forced spatial vibrations, caused by unbalance of the cutting tool. The model is shown in Fig. 3.

The following symbols are used:

m_1, m_2 – mass of the wood shaper and its spindle;
$\mathbf{I_1}$, $\mathbf{I_2}$ – inertia moment tensors of the wood shaper and its spindle;
$\mathbf{c_{x1i}}, \mathbf{c_{y1i}}, \mathbf{c_{z1i}}$, $i = 1, 2, 3, 4$ – elastic coefficients of the vibroisolators between the machine and the floor;
$\mathbf{b_{x1i}}, \mathbf{b_{y1i}}, \mathbf{b_{z1i}}$, $i = 1, 2, 3, 4$– damping coefficients of the vibroisolators between the machine and the floor;
$\mathbf{c_{x2i}}, \mathbf{c_{y2i}}, \mathbf{c_{z2i}}$, $i = 5, 6$ – elastic coefficients between the machine and the spindle;
$\mathbf{b_{x2i}}, \mathbf{b_{y2i}}, \mathbf{b_{z2i}}$, $i = 1, 2, 3, 4$ – damping coefficients between the machine and the spindle .

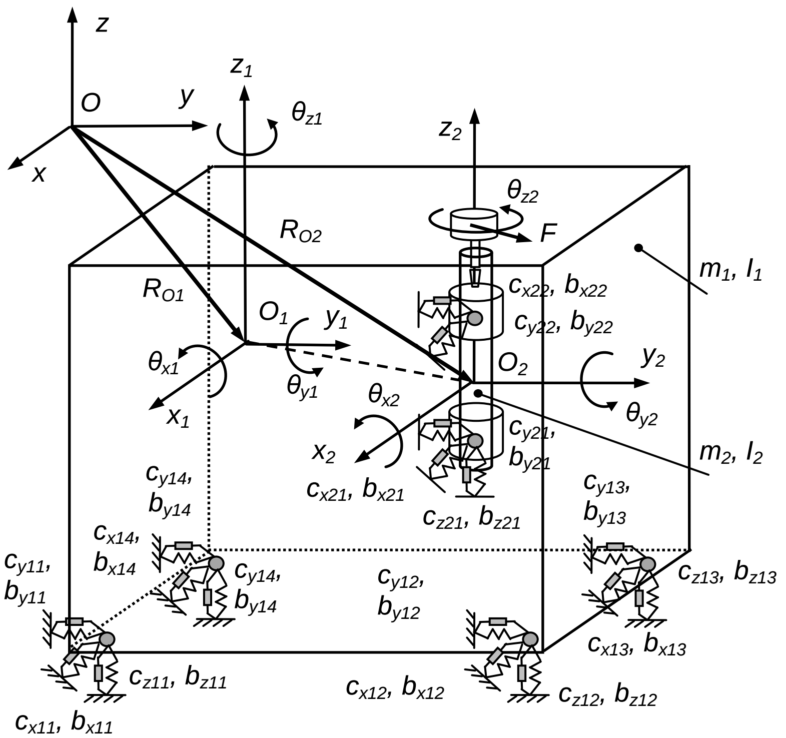

Fig. 3.
Mechanic-mathematical model of the wood shaper and its spindle.

The vector of the generalized coordinates is (Fig. 3)

$$q = \left[x_1 \; y_1 \; z_1 \; \theta_{x1} \; \theta_{y1} \; \theta_{z1} \; x_2 \; y_2 \; z_2 \; \theta_{x_2} \; \theta_{y_2} \; \theta_{z_2} \right]^T . \qquad (1)$$

The matrixes of the transition in small vibrations between the local coordinate systems of the bodies and the reference coordinate system have the form

$$A_i^0 = \begin{bmatrix} 1 & -\theta_{zi} & \theta_{yi} & x_i \\ \theta_{zi} & 1 & -\theta_{xi} & y_i \\ -\theta_{yi} & \theta_{xi} & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad i = 1,2 . \qquad (2)$$

The vector of the position of the center of mass of the relevant body is determined with

$$R_{Ci}^0 = A_i^0 . r_{Ci} = \begin{bmatrix} l_{Cx} + x_i + l_{Cz} . \theta_{yi} - l_{Cy} . \theta_{zi} \\ l_{Cy} + y_i - l_{Cz} . \theta_{xi} + l_{Cx} . \theta_{zi} \\ l_{Cz} + z_i + l_{Cy} . \theta_{xi} - l_{Cx} . \theta_{yi} \\ 1 \end{bmatrix} \qquad i = 1,2 . \qquad (3)$$

where: $r_{Ci} = \left[l_{Cx} \quad l_{Cy} \quad l_{Cz} \right]^T$ is the vector of the position of the center of mass in the local coordinate system.

The vector of absolute linear velocity of the center of mass of the respective body is calculated as follows

$$V_{Ci}^{0}=\frac{dR_{Ci}^{0}}{dt}=\begin{bmatrix}\dot{x}_i+l_{Cz}.\dot{\theta}_{yi}-l_{Cy}.\dot{\theta}_{zi}\\ \dot{y}_i-l_{Cz}.\dot{\theta}_{xi}+l_{Cx}.\dot{\theta}_{zi}\\ \dot{z}_i+l_{Cy}.\dot{\theta}_{xi}-l_{Cx}.\dot{\theta}_{yi}\\ 0\end{bmatrix}\qquad i=1,2. \tag{4}$$

The vector of absolute angular velocity of the respective body, projected in the local coordinate system, has the form

$$\Omega_i^i=\begin{bmatrix}\dot{\theta}_{xi}\\ \dot{\theta}_{yi}\\ \dot{\theta}_{zi}\\ 0\end{bmatrix}\qquad i=1,2. \tag{5}$$

The deduction of the kinetic energy and potential energy of the system is convenient to be made with a symbolic method and modern software (Mathematica, MATLAB).
The kinetic energy of the mechanical system is determined with

$$E_K=\sum_{i=1}^{2}\left(\frac{1}{2}.\left[\dot{R}_{Ci}^{\ T}\quad \dot{\Theta}_i^{\ T}\right]^T.\begin{bmatrix}m_{RRi} & \\ & I_{\Theta\Theta i}\end{bmatrix}.\begin{bmatrix}\dot{R}_{Ci}\\ \dot{\Theta}_i\end{bmatrix}\right), \tag{6}$$

where $m_{RRi}=\begin{bmatrix}m_i & 0 & 0\\ 0 & m_i & 0\\ 0 & 0 & m_i\end{bmatrix}$; $\mathbf{I}_{\Theta\Theta i}=\begin{bmatrix}I_{xxi} & -I_{xyi} & -I_{xzi}\\ -I_{xyi} & I_{yyi} & -I_{yzi}\\ -I_{xzi} & -I_{yzi} & I_{zzi}\end{bmatrix}$;

$\dot{R}_{Ci}=\left[\dot{x}_{Ci}\quad \dot{y}_{Ci}\quad \dot{z}_{Ci}\right]^T$; $\dot{\Theta}_i=\left[\dot{\theta}_{xi}\quad \dot{\theta}_{yi}\quad \dot{\theta}_{zi}\right]^T$.

Potential energy is defined by

$$E_P=E_{P1}+E_{P2}=\left(\sum_{k=1}^{4}\frac{1}{2}c_k.\left(\delta r_k^{01}\right)^2+\sum_{k=1}^{2}\frac{1}{2}c_k.\left(\delta r_k^{12}\right)^2\right)+\sum_{i=1}^{2}E_{PGi}, \tag{7}$$

where $\delta r_k^{01}=R_1+U_1^0.r_k^{01}-r_k^{01}$,
$\delta r_k^{12}=\left(R_1+U_1^0.r_k^{12}-r_k^{12}\right)-\left(R_2+U_2^0.r_k^{21}-r_k^{21}\right)$,
$R_i=\left[x_i\quad y_i\quad z_i\right]^T$ $i=1,2$ - vector of the position of the beginning of the mobile (related with the body) coordinate system relative to the fixed coordinate system,
δr_k^{01} – the deformation of the elastic elements between the base (marked conditionally with "0") and the body 1,
δr_k^{12} – the elastic deformation of the elements between the two bodies.

The differential equations which describe the free vibrations are deduced by using the Lagrange's method. This method provides the best opportunities.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right)-\left(\frac{\partial E_K}{\partial q}\right)+\frac{\partial F_b}{\partial \dot{q}}+\frac{\partial E_P}{\partial q}=Q, \tag{8}$$

where: E_K and E_P are respectively the kinetic and the potential energy of the systems. F_b is the energy dissipation or dissipative function. Q is the vector of generalized forces.

The obtained system of differential equations, which describes the forced spatial vibrations of the mechanical system, is

$$M_{12\times12}.\ddot{q}_{12\times1}+B_{12\times12}.\dot{q}_{12\times1}+C_{12\times12}.q_{12\times1}=Q_{12\times1}. \quad (9)$$

The matrix in these equations which characterizes the mass-inertial properties of the mechanical system is *M***,** and the elastic properties – *C***.** $B(\dot{q})$ is the matrix that characterizes the damping properties of this system and *Q* presents generalized forces

$$M=\left[a_{ij}\right], \qquad a_{ij}=\frac{\partial^2 E_K}{\partial\dot{q}_i\,.\,\partial\dot{q}_j}, \quad (10)$$

$$C=\left[c_{ij}\right], \qquad c_{ij}=\frac{\partial^2 E_P}{\partial q_i\,.\,\partial q_j}. \quad (11)$$

The matrix $B=\left[b_{m,n}\right]$ is obtained by substituting the elements of the matrix C – $c_{m,n}$, with $b_{m,n}$.

$$B=\left[b_{ij}\right], \qquad b_{ij}=\frac{\partial^2 F_b}{\partial\dot{q}_i\,.\,\partial\dot{q}_j}. \quad (12)$$

The vector of generalized external forces has the form

$$Q=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & Q_{F(3\times1)}{}^T & Q_{Q(3\times1)}{}^T\end{bmatrix}^T. \quad (13)$$

where

$$Q_F=\begin{bmatrix}F.cos(\omega.t)\\ F.sin(\omega.t)\\ 0\end{bmatrix}, \quad (14)$$

$$Q_Q(F)=U_i^{\Omega0^T}.\left(\tilde{r}_{Pi}^{0^T}.Q_F\right), \quad (15)$$

$$U_i^{\Omega0^T}=\begin{bmatrix}1 & 0 & \theta_{y1}\\ 0 & 1 & -\theta_{x1}\\ 0 & \theta_{x1} & 1\end{bmatrix}^T, \quad (16)$$

$$\tilde{r}_{Pi}^{0^T}=\begin{bmatrix}0 & l_{Piz}^0 & -l_{Piy}^0\\ -l_{Piz}^0 & 0 & l_{Pix}^0\\ l_{Piy}^0 & -l_{Pix}^0 & 0\end{bmatrix}, \quad (17)$$

The receiving of general solutions of the system (9) is connected with the determination of the initial conditions of motion *q (0)* and $\dot{q}(0)$. These initial conditions depend on the type of motion of the system.

The general solutions of the system of differential equations, written in a matrix form in harmonious kind of disturbing forces and initial conditions $t=0, q(0)=q_0, \dot{q}(0)=\dot{q}_0$, are

$$\begin{aligned}q(t)=&\sum_{r=1}^{12}\frac{2}{g_r^2+h_r^2}[G_rM\dot{q}(0)+(-\alpha_rG_rM+\beta_rH_rM\ +G_rB)q(0)]\,.e^{-\alpha_rt}.cos\,\beta_rt+\\&+\sum_{r=1}^{12}\frac{2}{g_r^2+h_r^2}[H_r.M.\dot{q}(0)+(-\alpha_r.H_r.M-\beta_r.G_r.M+H_r.B).q(0)]\,.e^{-\alpha_rt}.sin\,\beta_rt+\\&+Re\{\sum_{k=0}^{n}\sum_{r=1}^{12}\frac{2}{g_r^2+h_r^2}\frac{\alpha_r.G_r+\beta_r.H_r+i.k.\Omega.G_r}{\omega_r^2-k^2.\Omega^2+i.2.k.\sigma_r.\omega_r.\Omega}Q.e^{ik\Omega t}\}\end{aligned} \quad (18)$$

where

$$g_r = -2\alpha_r \left(V_r^T.M.V_r - W_r^T.M.W_r\right) - 4\beta_r V_r^T.M.W_r + V_r^T.B.V_r - W_r^T.B.W_r;$$
$$h_r = 2\beta_r \left(V_r^T.M.V_r - W_r^T.M.W_r\right) - 4\alpha_r V_r^T.M.W_r + 2V_r^T.B.W_r;$$
$$G_r = g_r L_r + h_r R_r; \quad L_r = V_r.V_r^T - W_r.W_r^T;$$
$$H_r = h_r L_r - g_r R_r; \quad R_r = V_r.W_r^T + W_r.V_r^T. \tag{19}$$

CONCLUSIONS

This study presents a mechanic - mathematical model of a wood shaper and its spindle, developed by the authors. The model is designed for investigation of the forced spatial vibrations of this type of machinery, caused by unbalance of the cutting tool. It takes into account the characteristics in the construction of wood shapers. In this model the wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic and damping elements with each other and with the motionless floor. The model takes into account the necessary mass, inertia, elastic and damping properties of the elements of the considered system. It includes all needed geometric parameters of this system. A necessary system of matrix differential equations is compiled and analytical solutions are presented. Numerical investigations are carried out with them by using the parameters of a real machine. These investigations are presented in the next part of this study.

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