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# DETERMINATION OF CRITICAL ROTATIONAL SPEED OF SAW BLADES BY USING VARIOUS METHODS

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***Abstract***

*Lateral oscillation of saw discs is quite an unwished situation and causes adverse effects not only on the work piece but also on the tool. The paper deals with the problem of circular saw blade oscillation and with the effect of modifying the natural frequency and critical rotational speed of the circular saw blade body. A calculation and experimental investigation of natural frequencies and node shapes was carried out on two types of circular saw blades and two methods for determination of natural frequencies are presented. First, a theoretical method – modal analysis (Finite Element Method, next FEM), and second, an experimental measurement. The results of investigation were used in order to compare the FEM and experimental methods and to show which modification achieved higher critical rotational speed. Both methods were done on two circular saw blades during continual cutting with 36 number of teeth and slots. Differences between calculated critical revolutions ranged from 0.5 % to 5.9%, depending on the number of nodal diameter.*

***Key words:*** *circular saw blade; critical rotational speed; modal analysis; natural frequencies; slots.*

## INTRODUCTION

Circular discs are very wide-spread parts or elements of machines, used in technological processes. Among others, we can mention the magnetic medium in floppy diskettes, compact discs, hard discs, discs of brakes, gas turbines or circular saw discs. The last mentioned are probably the most spread tools in the wood industry (used for splitting wood and wood based material) or metal industry (e.g. for dividing pipes, semi-finished shapes etc.)

A continual growth of the price of wood material is a reason why the circular saw blades producers aim at producing the blades thinner and thinner. Although such a way of increasing the economic effectiveness is to be considered praiseworthy, there are certain limitations with regard to the mechanic qualities of the tool and its behaviour during rotation. A thin-wall circular board with a hole of a given perimeter and rotating in the physical environment of the air (which a circular saw blade certainly is) can lose its rigidity when rotated – i.e. it can collapse because of the loss of dynamical stability and the resonance arisen. Furthermore, the loss of rigidity results in the cutting slot being widened (and the wood material being lost), in some cases even in a fatal destruction of the circular saw blade; for instance, caused by the circular saw blade having run into the sawn material and the axial forces being increased. Another factor possibly leading to the instability of the circular saw blade is the effort of the producers to cut grooves of various shapes into the body of the blade with the aim to decrease the noise of sawing. However, when located unsuitably, the grooves can cause more troubles than improvement

A lot of researchers analysed and presented results of their investigations. Some of them are thereinafter mentioned. Southwell (1922) reported a study on free vibration of flexible discs. They presented a theoretical study of a general gyroscope system and applied it to a spinning disc. Liang *et* *al*. (2002) studied the free and forced vibrations of a rotating polar orthotropic annual plate with a stationary concentrated transverse load. They used Galerkin approximation to evaluate the eigenvalues of the system and stressed that discs with higher modulus ratios or poison's ratios have higher natural frequencies. Ferretti *et al*. (2002) presented a methodology based on dynamic modelling and experimental measurements to study the vibrations in hard disc drives. Wang (2005) determined the natural frequencies of a circular plate with an attached core for different boundary conditions. Koo (2006) calculated the natural frequency and critical speed for rotating composite discs by the Rayleigh–Ritz method. He showed that the circumferentially-reinforced disc is more effective in increasing critical speed than the radially-reinforced disk.

Bauer and Eidel (2007) determined the lower approximate natural frequencies of a spinning circular plate for various boundary conditions. They determined the approximate lower natural frequencies as functions of the speed of spin. Duan *et al*. (2008) found out that fundamental vibration modes of circular plates with free edges can be modified by increasing the bending rigidity of the outer rim of the circular plate using a larger thickness or by using a material with a larger Young's modulus; or both. Bashmal *et al*. (2009) studied the in-plane modal characteristics of circular annular discs under combinations of all possible classical boundary conditions. The phenomenon of oscillation and its determination by experimental methods was dealt by Mote (1965); Stakhiev (2000, 2003); Schajer and Mote (1983); Schajer (1992); Yu and Mote (1987); Svoreň (1986); Nishio and Marui (1996); Orlowski et al., (2007); Veselý et al., (2012). Application of the finite element method for solving oscillation of saw blades was researched by Gogu (1988); Leopold and Münz (1992); Michna and Svoreň (2007); Ekevad et al. (2009); Cristóvão et al. (2012); Droba et al. (2013).

Studies on vibration of circular plates (discs) subjected to moving loads were initiated in the 1970s. They were meant to represent computer floppy discs and wood saws. Mote (1965) first studied the vibration of a disc modelled as a thin, flat, circular Kirchhoff plate subjected to a moving mass. Mote and his colleagues have published numerous papers on the vibration of different disc models under various moving loads.

(http://www.onlinetoolreviews.com)

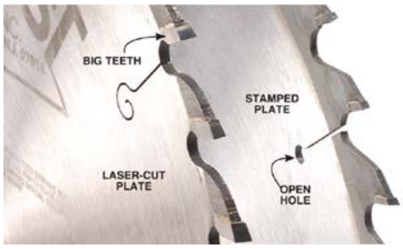


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***Fig.1.***
***Various shapes of slots in the saw disc's body***

Fundamental considerations and derivation of equations of motion are based on Kirchhoff's assumptions. The assumptions are valid only for thin circular discs. The field of displacements in the cylindrical coordinate's (r, φ, z) using Kirchhoff plate theory can be written in following form (in Cartesian rectangular coordinates):

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\cdot\frac{\partial^4 w}{\partial x^2\cdot\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=-q(x,y,t)-2\cdot a\cdot\rho\cdot\frac{\partial^2 w}{\partial t^2} \tag{1}$$

For a plate with constant thickness 2a:

$$D=\frac{2\cdot E\cdot a^3}{3\cdot(1-\nu^2)} \tag{2}$$

where: D – bending stiffness of the plate, in N·m;
E – modulus of elasticity, in $N\cdot m^{-2}$ (=2.1· $10^{11}$);
2a – thickness of the circular saw blade, in (mm);
ν – Poisson ratio, (=0.33);
ρ – density, in ($kg\cdot m^{-3}$) (=7800);
t – time, in sec.

Deflection of circular saw blade is expressed by (Nishio and Marui 1996) as:

$$w_{(r,\varphi,t)} = f_{(r)} \cdot \sin(k \cdot \varphi) \cdot \cos(2\pi \cdot f_n \cdot t) \tag{3}$$

$$w_{\ (r,n,t)} = \frac{f_{(r)}}{2} \cdot \sin 2\pi \cdot (f_n + k \cdot n) \cdot t - \frac{f_{(r)}}{2} \cdot \sin 2\pi \cdot (f_n - k \cdot n) \cdot t \tag{4}$$

where:
w – deflection of the circular saw blade, in m;
r,φ *t*– polar coordinate system, in m, in deg;
k – number of nodal diameters,
n – rotational speed of the saw disc, in $sec^{-1}$;
$f_n$ – natural frequency of the non-rotating circular saw blade, in Hz,
$f_{(r)}$ – function defining the deflection in radial direction.

Inaccurate cut, low quality of the surface, and high level of noise during sawing are the main problems of cutting on circular saws. These adverse effects are related to oscillation of circular saw blade (next marked CSB1, CSB2).

Nowadays, high requirements are imposed on cutting with circular saw blades, such as high surface quality, straight cut, reducing noise level etc. For meeting these requirements it is necessary to use high rotational speed. But then, a problem with critical rotational speed and instability of the circular saw blade arises. During the cutting process, oscillation is emitted in the circular saw blade which may lead to its instability. The reduction of the amplitude of oscillations is essential for reducing the adverse effects.

For determination of resonant and critical rotational speeds, the theory of oscillation is used: the oscillation of the circular saw blade is a superposition of two moving waves travelling in opposite direction to each other (forward and backward travelling wave). Frequency of these waves can be expressed as follows:

… for the forward travelling wave:

$$f_1 = f_{dyn(n)} + k \cdot n \quad [Hz] \tag{5}$$

… for the backward travelling wave:

$$f_2 = f_{dyn(n)} - k \cdot n \quad [Hz] \tag{6}$$

where:
$f_{dyn(n)}$ – frequency of the rotating circular saw blade, in Hz;
$f_1$ – frequency of the forward travelling wave, in Hz;
$f_2$ – frequency of the backward travelling wave, in Hz.

If the rotational speed of the circular-saw blade is increased, the frequency of the backward travelling wave becomes zero at certain rotational speed (except of the nodal diameters k = 0 and 1). This working speed is called "critical rotational speed". At this critical rotational speed, the angular speed of the circular-saw blade equals to the speed of the wave in the circular-saw blade and the backward travelling wave appears as if stopped in space. This is a resonance point where even a small lateral force causes large lateral deflection of the circular saw blade (Stakhiev 1970). From the equation (2) it is possible to derive the following one:

$$n_k = \frac{f_{dyn(n)}}{k} \ (rp\sec) \tag{7}$$

As a result of centrifugal force, natural frequency of the rotating circular saw blade increases parabolically with the increasing operating speed. The relationship between the natural frequency of the rotating circular saw blade and the rotation speed is expressed in the equation:

$$f_{dyn(n)}^2 = f_n^2 + \lambda \cdot n^2 \ (Hz) \tag{8}$$

where:
$\lambda$ – coefficient of the centrifugal force.

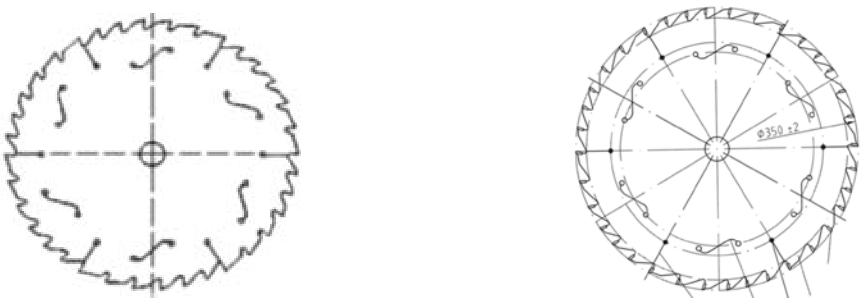
This critical rotational speed can be expressed by substituting the equation (8) into the equation (7) so that the following equation (9) arises (Nishio and Marui 1996; Schajer and Mote 1983; Svoreň 1986):

$$n_k = \frac{f_n}{\sqrt{k^2 - \lambda}} \quad (rp\sec) \tag{9}$$

The aim of this experiment was to find such design solutions which could increase dynamical stiffness of saw discs without increasing their thickness. The dynamical stiffness may be evaluated indirectly, searching for critical rotational speed, i.e. rotational speed during the saw lost stability.

## MATERIALS AND METHODS

Two circular saw blades were used for experimental measurements. There were cut slots (sometimes called compensation) In the body of both, in the first circular saw blade (CSB1) slots were inclined, from the radial direction, at an angle of 45°; in the second at an angle of 60°. The material of teeth plates was tungsten carbide and the teeth were alternately skewed, i.e. angle $\kappa_r \neq 90^o$ The used clamping collar had the external diameter $d_p$ = 110mm; the clamping ratio $\alpha$ was 0.314. Design differences of the circular saw blades are shown in Fig. 2. More detailed parameters about the discs are mentioned in Table 1. Teeth of the discs were in six identical groups, but the pitch between the teeth was not constant.




***a) Saw disc CBS1*** ***b) Saw disc CBS2***

***Fig. 2.***
***Construction of circular saw blades used for experimental measuring***

***Table 1***

***Parameters of the circular saw blades***

| | | |
|---|---|---|
| Diameter of the cutting circle (mm) | | 350 |
| Diameter of the clamping hole (mm) | | 30 |
| Number of teeth (-) | | 36 |
| Thickness of the body (mm) | | 2.4 |
| Height of the tooth (mm) | | 13 |
| Geometry of the tooth: | $\alpha_f$ ($^o$) | 15 |
| | $\beta_f$ ($^o$) | 65 |
| | $\gamma_f$ ($^o$) | 10 |
| Type of the tooth | | WZ |

The methods of measurement were based on the theory of vibration of the rotating circular blade.

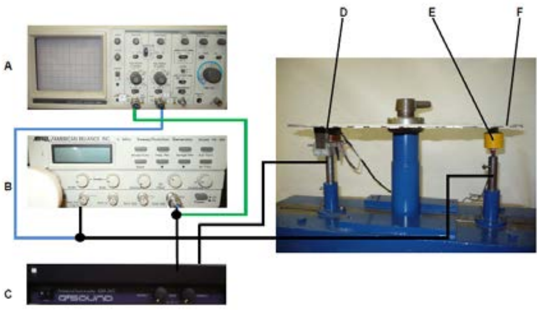
Two methods were used in order to determine the natural frequencies of the chosen circular saw blades:

- experimental measuring,
- Finite Element Method.

### Experimental measuring

The experimental measuring was carried out in the labs of the Department of woodworking machines and equipment of the Technical University in Zvolen. It consisted of two consecutive steps:

A. Assignation of the natural frequencies $f_n$ of the non-rotating circular-saw blade for k = 1, 2, 3, 4 on the device showed in Fig. 3.

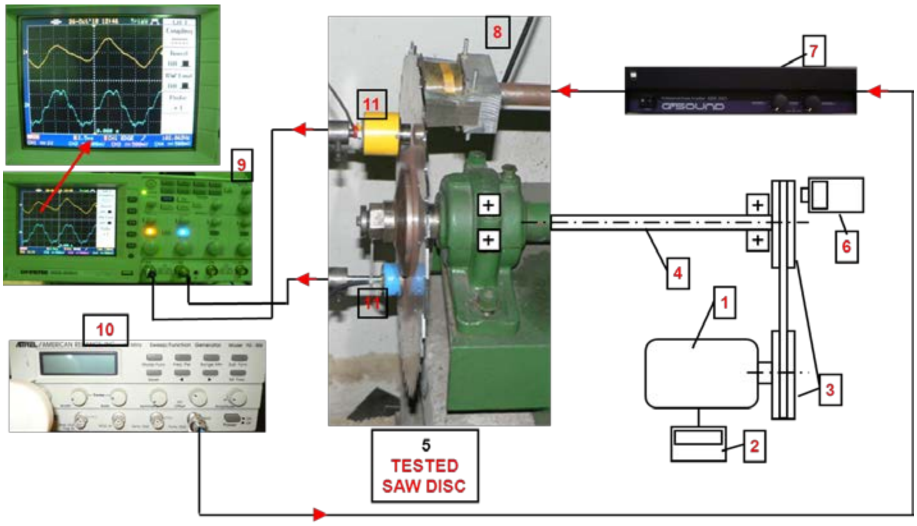



A - oscilloscope, B - tone generator, C – amplifier, D - Electromagnetic (solenoid) driver, E – non-contact displacement transducer, F - circular saw blade

***Fig. 3.***

***Equipment for experimental measuring of the frequency $f_n$***

B. Assignation of the centrifugal coefficient, the frequencies $f_2$ of the backward travelling waves for the same **k,** and the clamping ratio $\alpha$ on the measuring stand showed in Fig. 4.




1 – electric motor, 2 – frequency converter, 3 – belt drive, 4 – arbor in the bearings, 5 – circular saw blade mounted with clamping collars, 6 – noncontact speedometer, 7 – amplifier, 8 – electromagnetic driver, 9 – digital oscilloscope, 10 – frequency generator, 11 – sensors of vibration

***Fig. 4.***

***Equipment for experimental measuring of the frequency $f_2$***

**Finite Element Method**

As for the theoretical experiment, the software Pro/Engineer WF4 was used, which allows to perform a simulation of static and dynamic modal analyses. The software made use of FEM. There were created modal analyses on models of circular saw blades. These models were created according to drawing of real circular saw blades (Fig. 2a and 2b).

Both models of circular saw blades were clamped by using the function constrain displacement with a diameter of 110mm. Thanks to clamping collars, the area was absolutely rigid. There was done idealization on both saw blades, the model was defined as a thin shell disc. The mesh for the analysis was made from shell elements with the maximal size of 5mm (Fig. 5). Scrap view on the right side shows the mesh in detail as well as the interconnection of finite elements; the symbols around the saw disc represents the mass of the cutting plate (here, the tungsten carbide), concentrated at the pinpoints of the saw disc teeth. The chosen type of mesh was selected because of higher accuracy of calculated results.

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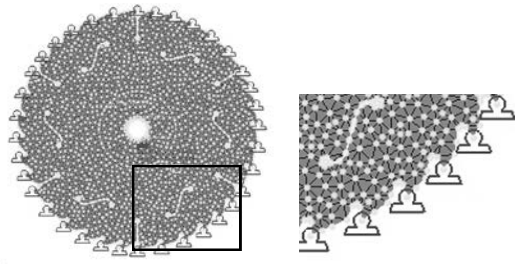

***Fig. 5.***
***Mesh used for modal analyses for both saw disc***

From the modal analyses, calculated values of natural frequencies and graphic results of the displacement of circular saw blades for nodal diameter k = 1, 2, 3, 4 were obtained.

**RESULTS AND DISCUSSION**

The values of natural frequencies of the cosine and sine components of the split modes (Yu and Mote 1987) are shown in tables; Table 2 displays experiment results, Table 4 modal analysis results.

***Table 2***

***Experimentally measured values of natural frequencies of CSB1, CSB2***

| **k** | **1** | | **2** | | **3** | | **4** | |
|---|---|---|---|---|---|---|---|---|
| | cosine | sine | cosine | sine | cosine | sine | cosine | sine |
| | Frequency $f_{n(EXP)}$[1] (Hz) | | | | | | | |
| **CSB1** | 105.9 | 105.9 | 142.7 | 142.7 | 235.2 | 240.5 | 381.1 | 381.1 |
| **CSB2** | 107.2 | 107.2 | 137.2 | 137.2 | 221.6 | 228.2 | 356.2 | 356.2 |

***Table 3***

***Calculated values of the coefficient of centrifugal force and critical rotational speed from experimentally measured values of the natural frequencies CSB1, CSB2***

| K | 2 | | | | 3 | | | | 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Type | CSB1 | | CSB2 | | CSB1 | | CSB2 | | CSB1 | | CSB2 | |
| Component of the split mode | cosine | sine | cosine | sine | cosine | sine | cosine | sine | cosine | sine | cosine | sine |
| $\lambda_{(EXP)}$ | 1.91 | 1.91 | 1.99 | 1.99 | 2.44 | 2.67 | 2.85 | 2.91 | 3.79 | 3.79 | 3.94 | 3.94 |
| $n_{k\ (EXP)}$ (rpm) | 5928 | 5928 | 5800 | 5800 | 5506 | 5734 | 5362 | 5550 | 6544 | 6544 | 6154 | 6154 |

The values of $\lambda_{(EXP)}$ displayed in Tab. 3 were computed using formulas (6) and (8) on the basis of the values of $f_2$, which were obtained experimentally (see Fig.4).

The values of $n_{k(EXP)}$ were calculated by the formula (9) on the basis of the values of $f_n$, which were obtained experimentally using the equipment in Fig. 3 and values $\lambda_{(EXP)}$ displayed in Tab. 3.

***Table 4***

***Values of the static natural frequencies of CSB1 and CSB2 calculated by modal analysis***

| k | 1 | | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|---|---|
| | Cosine | sine | cosine | sine | cosine | sine | Cosine | sine |
| | Frequency $f_{n\ (FEM)}$[2] (Hz) | | | | | | | |
| CSB1 | 135.5 | 135.5 | 157.4 | 157.4 | 224.8 | 236.2 | 347.1 | 347.1 |
| CSB2 | 137.4 | 137.4 | 159.6 | 159.6 | 225.4 | 235.2 | 353 | 353 |

[1] Inferior index EXP means, that values were obtained experimentally or computed from values obtained experimentally.
[2] Inferior index FEM means, that the values were obtained by FEM or computed from values obtained by FEM.

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***Table 5***

***Values of the coefficient λ of the centrifugal force and critical rotational speed calculated from the natural frequencies obtained by FEM***

| k | 2 | | 3 | | 4 | |
|---|---|---|---|---|---|---|
| Type | CSB1 | CSB2 | CSB1 | CSB2 | CSB1 | CSB2 |
| Component of the split mode | cosine | cosine | cosine | cosine | Cosine | cosine |
| $\lambda_{(FEM)}$ | 2.13 | 2.17 | 2.87 | 2.90 | 3.78 | 3.91 |
| $n_{k\ (FEM)}$ (rpm) | 6913.8 | 7071.1 | 5449.4 | 5477.1 | 5958.7 | 6091.1 |

The values of $\lambda_{(FEM)}$ mentioned in Table 5 were calculated from formula (8) on the basis of the values of $f_{(dyn)n}$, which were obtained by dynamic modal analysis and established for 3600 rpm (i.e. angular velocity 376.8 per sec).

The values of $n_{k(FEM)}$ were calculated from formula (9) on the basis of the values of $f_{n\ (FEM)}$ (displayed in Tab. 4), which were obtained by static modal analysis and values of $\lambda_{(FEM)}$ displayed in Tab. 5.

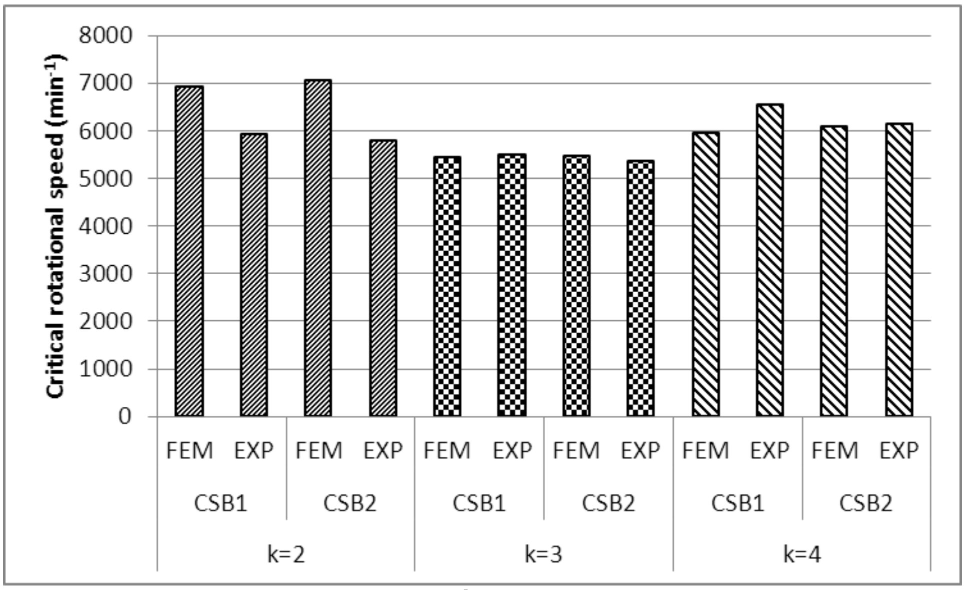


***Fig. 6.***

***Influence of nodal diameters number, saw disc type, and methodology on the critical revolutions***

After analysing the values displayed in Fig. 6, it is possible to comment the outcomes from two points of view:

1. with regard to the saw disc type (CSB1 and CSB2) and used method (FEM or EXP) and their influence on the evaluation of critical revolutions, i.e. the influence of slots slope on the critical revolutions for every number of nodal diameters <k> and used method
*(<u>example 1:</u> values for k=2, saw disc CSB1, method FEM compared with values for k=2, saw disc CSB2, method FEM; <u>example 2:</u> values for k=2, saw disc CSB1, method EXP compared with values for k=2, saw disc CSB2, method EXP; etc., next k=3 and k=4).*

or

2. with regard to both methods (FEM or EXP) and their shared ability to evaluate critical revolutions for every saw disc type (CSB1 and CSB2) and every number of nodal diameters <k>.
*(<u>example 3:</u> values for k=2, saw disc CSB1, method FEM compared with values for k=2, saw disc CSB1, method EXP; <u>example 4:</u> values for k=2, saw disc CSB2, method FEM compared with values for k=2, saw disc CSB2, method EXP; etc., next k=3 and k=4).*

Ad1. If we compare the influence of the saw disc type (i.e. slope of slots) on critical revolutions, we may assert that their influence, i.e slots displacement from $45^o$ to $60^o$ is practically trifling and the results obtained by FEM are very similar to the results received experimentally.

The differences between both methods were as follows:

- for k=2, methods FEM / EXP were 157 rpm / 128 rpm, i.e. 2.2 % / 2.2%;
- for k=3, methods FEM / EXP were 28 rpm / 144 rpm, i.e. 0.5 % / 2.2%;
- for k=4, methods FEM / EXP were 132 rpm / 390 rpm, i.e. 2.5 % / 5.9%,

it means, that both methods are very good and their comparability is very high.

Ad2. If we compare the results of individual methods adherent to the same saw disc (for every <k>), we can state, that the biggest differences appear for k=2. According to FEM, the results (critical revolutions) are about 14,2% higher than critical revolution according to EXP for the saw disc CSB1; for the saw disc CSB2 there is a difference of 1271 rpm, i.e. almost 18%.

It is very interesting that for k=3, the accordance is almost 100% - precisely 99% for CSB1 and 98% for CSB2. In the group k=4, it is nearly 90% for CSB1 and 99% for CSB2, nevertheless, these differences are considerably smaller than the results for the saw disc with k=2.

## CONCLUSIONS

On the basis of the carried out experiments it can be stated that:

1. Neither of the applied methods confirmed positive influence of the saw disc body's design on its dynamical stiffness, which was indirectly proven by critical revolutions. The reason can be a small difference of the angle of swing out.

2. Good conformity of both methods can be a good argument for manufacturers to use available software that can relatively exactly simulate the process of the phenomenon – except for revolutions connected with 2 nodal diameters.

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