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DETERMINATION OF THE LIMITING STRESS STATE IN THE TIMBER WITH THE GRADIENT HUMIDITY FIELDS

Bogdan POBEREYKO
Prof.dr.eng. – Ukrainian National Forestry University in Lviv
Address: 103 Gen. Chuprynka st., 79057 Lviv, Ukraine
E-mail: messering@yandex.ru

Lubomyr FLUD
Teaching assistant – Ukrainian National Forestry University in Lviv
Address: 103 Gen. Chuprynka st., 79057 Lviv, Ukraine
E-mail: fludlybomir@gmail.com

Svitlana KOSHYRETS
Ph.D. in Engineering Science – Ukrainian National Forestry University in Lviv
Address: 103 Gen. Chuprynka st., 79057 Lviv, Ukraine
E-mail: Svitlana0981@mail.ru

Abstract:
An important task of hydrothermal technology of wood processing is the task of determining the stress tensor components distribution and their permissible values in the dry timber. After all, the high-quality dried material is considered to be the material in any part of which stress values do not exceed the permissible values. Today this problem is only partly solved. In general, mathematical models are developed in order to calculate the stress field in the timber with the uneven distribution of humidity fields in the elastic approach. Therefore, the research of the limiting stress state in the timber with the gradient humidity fields is of importance today.

Key words: *acceptable humidity range; strength criterion; tensor components strength.*

SETTING OF THE PROBLEM

It is known that, depending on the magnitude values of the mechanical, moisture or temperature loads the dry timber may exist in different states: elastic, viscoelastic, plastic, plastic-elastic and destructive. The set of all possible states is called the elastic region of the elastic deformation, the state of visco-elastic is called the region of viscoelastic deformation, etc. These regions are limited and defined by the respective terms of strength. Constraints for the elastic deformation region is, for example, the criteria of short-term strength (Pobereyko 2005):

$$F\left(\sigma_{ij}^{*}\right)=1, \quad i=1\div 3, \quad j=1\div 3 \tag{1}$$

where: σ_{ij}^{*} – tensor components strength.

Deformation of the material in different deformation regions are described by different mathematical models. Elastic region is described by the equations and the relations of the theory of elasticity; viscoelastic region is described by the viscoelasticity laws, etc. These models are limited. Their regions of determination are defined with the corresponding limitation of material deformation regions. For example, the equation

$$\varepsilon_{nm}=\varepsilon_{nm}\left(\sigma_{ij}\right) \tag{2}$$

describes the patterns of changes in strain tensor components ε_{nm} depending on the changes in the stress tensor components σ_{ij} only in the Cartesian stress space bounded by the appropriate strength condition.

Thus, the required mathematical model for determining the distribution of the strength tensor components (maximum stress state) in the timber with the uneven distribution of humidity is the mathematical model of calculating the stress state of the test material, supplemented by the appropriate strength condition.

MATHEMATICAL MODEL FOR CALCULATING THE STRESS FIELDS

At the stage of constant speed drying in tangential timber the stress state is flat and the value of the axial stress tensor component is almost negligible.

Therefore, in order to determine the strength tensor components in the dry timber we restrict ourselves with the mathematical model of calculating the stresses in the tangential board with plane stress state in the radial-tangential plane of the structural symmetry (Sokolovsky 1998). The components of this model in the elastic setting of the problem are:

- equilibrium equation

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} = \dfrac{\partial \tau_{xy}}{\partial y}; \\ \dfrac{\partial \tau_{xy}}{\partial x} = \dfrac{\partial \sigma_y}{\partial y}. \end{cases} \qquad (3)$$

- adjacency equation

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial xy}, \qquad (4)$$

- Hooke's law for incompressible and sensible materials

$$\begin{cases} \varepsilon_x = \dfrac{\sigma_x}{E_x} - \dfrac{\mu_{yx}\sigma_y}{E_y} + \beta_x W(x); \\ \varepsilon_y = -\dfrac{\mu_{xy}\sigma_x}{E_x} + \dfrac{\sigma_y}{E_y} + \beta_y W(x); \\ \gamma_{xy} = \dfrac{\tau_{xy}}{G_{xy}}. \end{cases} \qquad (5)$$

- boundary conditions

$\sigma_x = 0$ for $x = \pm R$; $\sigma_y = 0$ for $y = \pm a$; $\tau_{xy} = 0$ for $x = \pm R$ and $y = \pm a$ (6)

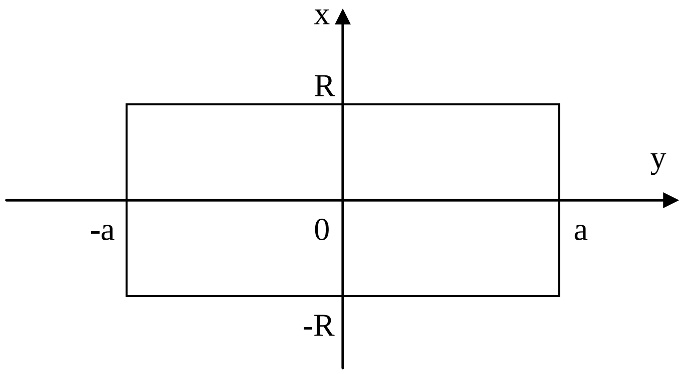

Fig. 1.
Cross-section of the tangential board.

Here: E_X, E_y – elastic moduli; G_{xy} – shear modulus; μ_{xy}, μ_{yx} – the Poisson ratios; β_x, β_y – shrinkage ratios; *W* – distribution function of relative timber humidity by its thickness; ε_x, ε_y – the main components of the strain tensor; τ_{xy}, γ_{xy} – stress and shear strain; *a* – half-width of the board; *R* – half-thickness of the board; x, y – coordinates of the cross-section of the board, shown in Fig. 1.

WOOD STRENGTH TERMS

According to (Pobereyko *et al*. 2013), the strength of wood of different species is described by different mathematical models. The elastic region of softwood is defined by the Goldenblata-Kopnov criteria (Seng 1994):

$$\Pi_{xx}\sigma_x + \Pi_{yy}\sigma_y + \sqrt{\Pi_{xxxx}\sigma_x^2 + \Pi_{yyyy}\sigma_y^2 + \Pi_{xxyy}\sigma_x\sigma_y + 4\Pi_{xyxy}\tau_{xy}^2} = 1 \qquad (7)$$

Here Π_{xx}, Π_{yy}, Π_{xxxx}, Π_{yyyy}, Π_{xxyy}, Π_{xyxy} – strength tensor components, which are determined by the formulas:

$$\Pi_{xx} = \frac{1}{2}\left(\frac{1}{\sigma_{xp}} - \frac{1}{\sigma_{xc}}\right); \quad \Pi_{yy} = \frac{1}{2}\left(\frac{1}{\sigma_{yp}} - \frac{1}{\sigma_{yc}}\right); \quad \Pi_{xyxy} = \frac{1}{4\left(\tau_{xy}^*\right)^2}$$

$$\Pi_{xxxx} = \frac{1}{4}\left(\frac{1}{\sigma_{xp}} + \frac{1}{\sigma_{xc}}\right)^2; \quad \Pi_{yyyy} = \frac{1}{4}\left(\frac{1}{\sigma_{yp}} + \frac{1}{\sigma_{yc}}\right)^2; \qquad (8)$$

$$\Pi_{xxyy}=\frac{1}{8}\left[\left(\frac{1}{\sigma_{xp}}+\frac{1}{\sigma_{xc}}\right)^2+\left(\frac{1}{\sigma_{yp}}+\frac{1}{\sigma_{yc}}\right)^2-\left(\frac{1}{\tau^*_{xy,45}}+\frac{1}{\overline{\tau}^*_{xy,45}}\right)\right];$$

$\sigma_{xp}, \sigma_{yp}, \sigma_{xc}, \sigma_{yc}$ – strength of wood tensile and compression in the directions OX and OY, τ^*_{xy} – wood strength, tested for pure shear.

The elastic region of hardwood is defined by the Y. Ashkenazi criteria (Seng 1994):

$$A_{xx}\sigma_x^2+A_{yy}\sigma_y^2+2A_{xxyy}\sigma_x\sigma_y+4A_{xyxy}\tau_{xy}^2=\sqrt{\sigma_x^2+\sigma_y^2+\sigma_x\sigma_y+\tau_{xy}^2}\,. \tag{9}$$

where:

$$A_{xx}=\frac{1}{\sigma_{xp}},\qquad A_{yy}=\frac{1}{\sigma_{yp}},\qquad A_{xyxy}=\frac{1}{4\tau^*_{xy}}, \tag{10}$$

$$A_{xxyy}=\frac{1}{\sigma_{xp}}+\frac{1}{\sigma_{yp}}-\frac{1}{4\tau^*_{xy,45}}$$

Thus, we have two mathematical models. One of them is given by the equations (3) – (8), and the other by the equations (3) – (6) and (9) – (10). The first is to determine the distribution of the limiting stress state fields in the softwood timber and the other one is to determine the distribution of the limiting stress state fields in the hardwood timber.

PRACTICAL IMPLEMENTATION OF THE MATHEMATICAL MODELS (3) – (8) AND (3) – (6) AND (9) – (10)

Equation (3) – (5) satisfy the functions (Sokolovsky 1998):

$$\sigma_x=\frac{\partial^2F(x,y)}{\partial y^2},\quad \sigma_y=\frac{\partial^2F(x,y)}{\partial x^2},\quad \tau_{xy}=-\frac{\partial^2F(x,y)}{\partial x\partial y}. \qquad \text{[MPa]} \tag{11}$$

Here $F(x,y)$ – any double-differentiated function of Erie, which is the solution of the equation:

$$\frac{1}{E_x}\frac{\partial^4F(x,y)}{\partial y^4}+\left(\frac{1}{G_{xy}}-\frac{\mu_{yx}}{E_y}-\frac{\mu_{xy}}{E_x}\right)\frac{\partial^4F(x,y)}{\partial x^2\partial y^2}+\frac{1}{E_y}\frac{\partial^4F(x,y)}{\partial x^4}=-\frac{\partial^2\beta_xW}{\partial y^2}-\frac{\partial^2\beta_yW}{\partial x^2} \tag{12}$$

received by substituting the equations (5) into (4), and with the subsequent replacement of σ_x, σ_y, τ_{xy} into $\frac{\partial^2F(x,y)}{\partial y^2}$, $\frac{\partial^2F(x,y)}{\partial x^2}$, $-\frac{\partial^2F(x,y)}{\partial x\partial y}$ respectively.

Thus, the problem of calculating the stress fields in the cross-section of the timber with uneven distribution of humidity is equivalent to the problem of finding the solution of equation (12) that satisfies the boundary conditions (6).

According to the studies (Sokolovsky 1998), this solution is function

$$F=d_1\varphi_1+d_2\varphi_2+d_3\varphi_3\,, \tag{13}$$

where: $\varphi_1, \varphi_2, \varphi_3$ – coordinate functions that are defined by the formulas:

$$\varphi_1=\left(x^2-R^2\right)^2\left(y^2-a^2\right)^2,\qquad \varphi_2=\varphi_1x^2,\qquad \varphi_3=\varphi_1y^2. \tag{14}$$

The unknown coefficients d_1, d_2, d_3 the so called Ritz coefficients are determined from the system of algebraic equations (2):

$$\sum_{k=1}^{3}\left(\Delta'\varphi_k,\varphi_n\right)d_k=\left(\left(-\frac{\partial^2\beta_xW}{\partial y^2}-\frac{\partial^2\beta_yW}{\partial x^2}\right),\varphi_n\right), \tag{15}$$

where:

$$\Delta'=\frac{1}{E_x}\frac{\partial^4}{\partial y^4}+\left(\frac{1}{G_{xy}}-\frac{\mu_{yx}}{E_y}-\frac{\mu_{xy}}{E_x}\right)\frac{\partial^4}{\partial x^2\partial y^2}+\frac{1}{E_y}\frac{\partial^4}{\partial x^4}.$$

Scalar product $\left(\Delta'\varphi_k,\varphi_n\right)$ and $\left(\left(-\frac{\partial^2\beta_xW}{\partial y^2}-\frac{\partial^2\beta_yW}{\partial x^2}\right),\varphi_n\right)$ are determined by the formulas:

$$\left(\Delta'\varphi_k,\varphi_n\right)=\int_{-a}^{a}\int_{-R}^{R}\left(\Delta'\varphi_k\right)\cdot\varphi_n\,dxdy\,; \tag{16}$$

$$\left(\left(-\frac{\partial^2 \beta_x W}{\partial y^2}-\frac{\partial^2 \beta_y W}{\partial x^2}\right),\varphi_n\right)=\int_{-a}^{a}\int_{-R}^{R}\left(-\frac{\partial^2 \beta_x W}{\partial y^2}-\frac{\partial^2 \beta_y W}{\partial x^2}\right)\varphi_n dxdy\,.$$

For timber with a parabolic distribution of humidity:

$$W(x)=\frac{\Delta W_{\max}}{R^2}x^2\,, \qquad [\%] \tag{17}$$

system of equations (15) has the form:

$$\begin{cases} B_{11}d_1+B_{12}d_2+B_{13}d_3=-\dfrac{512}{225}a^5R^3\beta_y\Delta W_{\max}; \\ B_{21}d_1+B_{22}d_2+B_{23}d_3=-\dfrac{512}{1575}a^5R^3\beta_y\Delta W_{\max}; \\ B_{11}d_1+B_{12}d_2+B_{13}d_3=-\dfrac{512}{1575}a^7R^3\beta_y\Delta W_{\max}, \end{cases} \tag{18}$$

where: $\Delta W_{\max}$ – the difference of relative humidity of central layer and the surface of the investigated timber;

$$B_{11}=\frac{32768}{1575}\left(\frac{a^5R^9}{E_x}+\frac{a^9R^5}{E_y}\right)+\frac{65536}{11025}\left(\frac{1}{G_{xy}}-\frac{\mu_{yx}}{E_y}-\frac{\mu_{xy}}{E_x}\right)a^7R^7\,;$$

$$B_{12}=B_{21}=\frac{32768}{1575}\left(\frac{a^5R^{11}}{11E_x}+\frac{a^9R^7}{7E_y}\right);$$

$$B_{13}=B_{31}=\frac{32768}{1575}\left(\frac{a^7R^9}{7E_x}+\frac{a^{11}R^5}{11E_y}\right); \tag{19}$$

$$B_{22}=\frac{32768}{1575}\left(\frac{a^5R^{13}}{143E_x}+\frac{a^9R^9}{7E_y}\right)+\frac{65536}{121275}\left(\frac{1}{G_{xy}}-\frac{\mu_{yx}}{E_y}-\frac{\mu_{xy}}{E_x}\right)a^7R^{11}\,;$$

$$B_{23}=B_{32}=\frac{32768}{121275}\left(\frac{a^7R^{11}}{E_x}+\frac{a^{11}R^7}{E_y}\right);$$

$$B_{33}=\frac{32768}{525}\left(\frac{a^9R^9}{7E_x}+\frac{a^{13}R^5}{143E_y}\right)+\frac{65536}{121275}\left(\frac{1}{G_{xy}}-\frac{\mu_{yx}}{E_y}-\frac{\mu_{xy}}{E_x}\right)a^{11}R^7\,.$$

The coefficients d_1, d_2, d_3 are dependent on the geometrical characteristics of the material and humidity range by its thickness. Indeed, according to (18):

$$d_1=-D_1\beta_y\Delta W_{\max}\,; \qquad d_2=-D_2\beta_y\Delta W_{\max}\,; \qquad d_3=-D_3\beta_y\Delta W_{\max}\,. \tag{20}$$

where:

$$D_1=\frac{\Delta_1}{\Delta}\,; \qquad D_2=\frac{\Delta_2}{\Delta}\,; \qquad D_3=\frac{\Delta_3}{\Delta}\,; \tag{21}$$

$$\Delta_1=\frac{512}{225}\begin{vmatrix} a^5R^3 & B_{12} & B_{13} \\ \dfrac{a^5R^3}{7} & B_{22} & B_{23} \\ \dfrac{a^7R^3}{7} & B_{32} & B_{33} \end{vmatrix}; \qquad \Delta_2=\frac{512}{225}\begin{vmatrix} B_{11} & a^5R^3 & B_{13} \\ B_{21} & \dfrac{a^5R^3}{7} & B_{23} \\ B_{31} & \dfrac{a^7R^3}{7} & B_{33} \end{vmatrix};$$

$$\Delta_3=\begin{vmatrix} B_{11} & B_{12} & a^5R^3 \\ B_{21} & B_{22} & \dfrac{a^5R^3}{7} \\ B_{31} & B_{32} & \dfrac{a^7R^3}{7} \end{vmatrix}; \quad \Delta=\begin{vmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{vmatrix}.$$

By substituting (20) and (14) to (13) and by the formulas (11) we define the stress tensor components:

$$\sigma_x=\beta_y\Delta W_{\max}\Phi_x\,; \qquad \sigma_y=\beta_y\Delta W_{\max}\Phi_y\,; \qquad \tau_{xy}=\beta_y\Delta W_{\max}\Phi_{xy}\,, \text{ [MPa]} \tag{22}$$

where:

$$\Phi_x=-2\left\{2\left(3y^2-a^2\right)\left(D_1+2D_2x^2\right)+D_3\left(15y^4-12a^2y^2+a^4\right)\right\}\left(x^2-R^2\right)^2;$$

$$\Phi_y = -2\{2(3x^2 - R^2)(D_1 + 2D_3 y^2) + D_2(15x^4 - 12R^2x^2 + R^4)\}(y^2 - a^2)^2; \quad (23)$$
$$\Phi_{xy} = 8\{2D_1 x(x^2 - R^2)(y^2 - a^2) + D_2 xy(3x^4 - 4R^2x^2 + R^4)(y^2 - a^2) +$$
$$+ D_3 xy(3y^4 - 4a^2y^2 + a^4)(x^2 - R^2)\}.$$

We substitute formula (22) into the strength criteria (7) and (9). As a result of this substitution we obtain the following mathematical model:

$$\Delta W_{\partial on} = \frac{1}{\beta_y\left(\Pi_{xx}\Phi_x + \Pi_{yy}\Phi_y + \sqrt{\Pi_{xxxx}\Phi_x^2 + \Pi_{yyyy}\Phi_y^2 + \Pi_{xxyy}\Phi_x\Phi_y + 4\Pi_{xyxy}\Phi_{xy}^2}\right)} \text{ [\%]}; \quad (24)$$

$$\Delta W_{\partial on} = \frac{\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_x\Phi_y + \Phi_{xy}^2}}{\beta_y\left(A_{xx}\Phi_x^2 + A_{yy}\Phi_y^2 + 2A_{xxyy}\Phi_x\Phi_y + 4A_{xyxy}\Phi_{xy}^2\right)}. \quad \text{[\%]} \quad (25)$$

Formula (24) allows to determine the permissible value $\Delta W_{\partial on}$ of the value $\Delta W_{\max}$ for an arbitrary selected point (x, y) of the cross section of the softwood timber, and the formula (25) – the permissible value $\Delta W_{\partial on}$ of the value $\Delta W_{\max}$ for an arbitrary selected point (x, y) of the cross section of the hardwood timber. Besides, these formulas allow calculating the boundary values σ_x^*, σ_y^* and τ_{xy}^* and the stress tensor components in the above mentioned range point of dry material. In fact, it is sufficient in the formulas (22) to replace $\Delta W_{\max}$ for $\Delta W_{\partial on}$. As a result, we have:

– for the softwood timber:

$$\sigma_x^* = \frac{\Phi_x}{\Pi_{xx}\Phi_x + \Pi_{yy}\Phi_y + \sqrt{\Pi_{xxxx}\Phi_x^2 + \Pi_{yyyy}\Phi_y^2 + \Pi_{xxyy}\Phi_x\Phi_y + 4\Pi_{xyxy}\Phi_{xy}^2}}, \quad \text{[MPa] (26)}$$

$$\sigma_y^* = \frac{\Phi_y}{\Pi_{xx}\Phi_x + \Pi_{yy}\Phi_y + \sqrt{\Pi_{xxxx}\Phi_x^2 + \Pi_{yyyy}\Phi_y^2 + \Pi_{xxyy}\Phi_x\Phi_y + 4\Pi_{xyxy}\Phi_{xy}^2}}, \quad \text{[MPa] (27)}$$

$$\tau_{xy}^* = \frac{\Phi_{xy}}{\Pi_{xx}\Phi_x + \Pi_{yy}\Phi_y + \sqrt{\Pi_{xxxx}\Phi_x^2 + \Pi_{yyyy}\Phi_y^2 + \Pi_{xxyy}\Phi_x\Phi_y + 4\Pi_{xyxy}\Phi_{xy}^2}}; \quad \text{[MPa] (28)}$$

– for the hardwood timber:

$$\sigma_x^* = \frac{\Phi_x\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_x\Phi_y + \Phi_{xy}^2}}{A_{xx}\Phi_x^2 + A_{yy}\Phi_y^2 + 2A_{xxyy}\Phi_x\Phi_y + 4A_{xyxy}\Phi_{xy}^2}; \quad \text{[MPa] (29)}$$

$$\sigma_y^* = \frac{\Phi_y\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_x\Phi_y + \Phi_{xy}^2}}{A_{xx}\Phi_x^2 + A_{yy}\Phi_y^2 + 2A_{xxyy}\Phi_x\Phi_y + 4A_{xyxy}\Phi_{xy}^2}; \quad \text{[MPa] (30)}$$

$$\tau_{xy}^* = \frac{\Phi_{xy}\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_x\Phi_y + \Phi_{xy}^2}}{A_{xx}\Phi_x^2 + A_{yy}\Phi_y^2 + 2A_{xxyy}\Phi_x\Phi_y + 4A_{xyxy}\Phi_{xy}^2}. \quad \text{[MPa] (31)}$$

ANALYSIS OF THE MATHEMATICAL MODELS (26) – (28) AND (29) – (31) IN ORDER TO DETERMINE THE LIMITING STRESS STATE IN SOFTWOOD AND HARDWOOD TIMBER

To analyze the formulas (26) – (31) we determine the limiting stress state in the surface points and in the points of the dry timber central layer. In order to do this, in the formulas (22) – (23) we replace x by R and x by 0 respectively. Then, after simple mathematical transformations we obtain:

– for $x = R$:

$$\Phi_x = 0; \quad \Phi_{xy} = 0; \quad \Phi_y = -8R^2\{2D_3y^2 + D_2R^2 + D_1\}(y^2 - a^2)^2; \quad (32)$$

– for $x = 0$:

$$\Phi_x = -2\{2D_1(3y^2 - a^2) + D_3(15y^4 - 12a^2y^2 + a^4)\}R^4; \quad (33)$$

$$\Phi_y = -2\left\{D_2R^4 - 2R^2\left(D_1 + 2D_3y^2\right)\right\}\left(y^2 - a^2\right)^2; \qquad \Phi_{xy} = 0.$$

Hence, substituting (32) into (26) – (28) we obtain the formula for determining the limiting stress state in the points of reservoir timber:

– softwood timber:

$$\sigma_x^* = 0, \qquad \tau_{xy}^* = 0, \qquad \sigma_y^* = \frac{\Phi_y}{\Pi_{yy}\Phi_y + \sqrt{\Pi_{yyyy}\Phi_y^2}} = \frac{1}{\Pi_{yy} + \sqrt{\Pi_{yyyy}}} = \sigma_{yp}; \quad \text{[MPa]} \tag{34}$$

– hardwood timber:

$$\sigma_x^* = 0, \qquad \tau_{xy}^* = 0, \qquad \sigma_y^* = \frac{\Phi_y\sqrt{\Phi_y^2}}{A_{yy}\Phi_y^2} = \frac{1}{A_{yy}} = \sigma_{yp}. \ \text{[MPa]} \tag{35}$$

In order to derive the formula for calculating the boundary stress tensor components in the central board layer with uneven distribution of humidity we substitute (33) into (26) – (28). As a result, we have:

– for the softwood timer boards:

$$\sigma_x^* = \frac{\Phi_x}{\Pi_{xx}\Phi_x + \Pi_{yy}\Phi_y + \sqrt{\Pi_{xxxx}\Phi_x^2 + \Pi_{yyyy}\Phi_y^2 + \Pi_{xxyy}\Phi_x\Phi_y}}, \ \text{[MPa]} \tag{36}$$

$$\sigma_y^* = \frac{\Phi_y}{\Pi_{xx}\Phi_x + \Pi_{yy}\Phi_y + \sqrt{\Pi_{xxxx}\Phi_x^2 + \Pi_{yyyy}\Phi_y^2 + \Pi_{xxyy}\Phi_x\Phi_y}}, \ \text{[MPa]}$$

$$\tau_{xy}^* = 0; \ \text{[MPa]}$$

– for the hardwood timber boards:

$$\sigma_x^* = \frac{\Phi_x\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_x\Phi_y}}{A_{xx}\Phi_x^2 + A_{yy}\Phi_y^2 + 2A_{xxyy}\Phi_x\Phi_y}, \ \text{[MPa]} \tag{37}$$

$$\sigma_y^* = \frac{\Phi_y\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_x\Phi_y}}{A_{xx}\Phi_x^2 + A_{yy}\Phi_y^2 + 2A_{xxyy}\Phi_x\Phi_y}, \ \text{[MPa]}$$

$$\tau_{xy}^* = 0. \ \text{[MPa]}$$

From the analysis of the formulas (34) – (37) it follows that on the surface of the timber with the uneven distribution of humidity the limiting stress state is uniaxial tensile stress state, and in the core layer – biaxial stress state. At any point of the surface the value of the tangential stress tensor components are equal to the wood strength, tested on stretching in the tangential direction of the anisotropy, and the values of all other components equal to zero. In contrast, at the points of the central layer only the valid values of shear stresses equal to zero, and the valid values of the radial and tangential stress tensor components differ from zero.

CONCLUSIONS

The mathematical models are defined in order to determine the fields of limiting stresses in the tangential softwood and hardwood timber. It was found that the limiting stress state is uniaxial tensile stress state on the surface of timber with the uneven distribution of humidity fields and in the central layer it has biaxial stress state.

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